The Principle of conservation of linear Momentum.

Theorem:

If no resultant external force acts on a system of bodies or particles then the algebraic sum of momentum of the system in any direction is unaltered by the mutual action of the bodies or particles whatever the original condition of the system may be

Proof:-

let us consider a system of two particles A and B having masses m_1 and m_2 respectively.

Since the bodies of the system are mutually acting upon one another and external forces are absent, the force which a particle A exerts on B is equal and opposite to that which the particle exerts on A.

(By Newton's third law of motion)

let P_1 and P_2 be the magnitude of the resolved parts of these two forces in a given direction, each force acting for time 'δt'

$$\therefore \quad P_1 = -P_2$$

Let u_1 and u_1 be the initial velocities and v_1, v_2 be the final velocities of A and B resp. in the given direction for an interval of time t'.
Then the impulse on B and A are as follows.

$$\int_0^{t'} P_1\,dt \;\cancel{Q}= m_2(v_2 - u_2) \quad \text{— (1)}$$

$$\int_0^{t'} P_2\,dt = m_1(v_1 - u_1) \quad \text{— (2)}$$

$\because \; P_1 = -P_2$

$$\therefore \int_0^{t'} P_1\,dt = -\int_0^{t'} P_2\,dt$$

From (1) and (2)

$$m_2(v_2 - u_2) = -\{m_1(v_1 - u_1)\}$$

$$\Rightarrow m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{— (3)}$$

Similarly,
if we consider other pairs of bodies of the system, which ~~get~~ also mutually acting upon each other for the same time t' then we shall get equations like (3) for each such pair.
Adding all these equations finally get

$$\sum m_k u_k = \sum m_k u_k$$

Hence, the algebraic sum of two momentum of the system in any direction remains unaltered by the mutual action of the bodies of the system.